10.3 Apply Properties of Chords

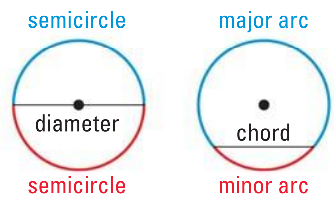
Before You used relationships of central angles and arcs in a circle.

Now You will use relationships of arcs and chords in a circle.

Why? So you can design a logo for a company, as in Ex. 25.

Key Vocabulary
- **chord,** *p. 651*
- **arc,** *p. 659*
- **semicircle,** *p. 659*

Recall that a *chord* is a segment with endpoints on a circle. Because its endpoints lie on the circle, any chord divides the circle into two arcs. A diameter divides a circle into two semicircles. Any other chord divides a circle into a minor arc and a major arc.

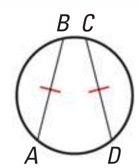


THEOREM *For Your Notebook*

THEOREM 10.3

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

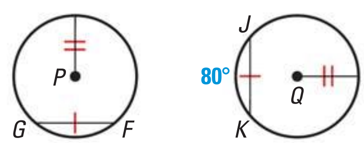
Proof: Exs. 27–28, p. 669



$\overset{\frown}{AB} \cong \overset{\frown}{CD}$ if and only if $\overline{AB} \cong \overline{CD}$.

EXAMPLE 1 Use congruent chords to find an arc measure

In the diagram, $\odot P \cong \odot Q$, $\overline{FG} \cong \overline{JK}$, and $m\overset{\frown}{JK} = 80°$. Find $m\overset{\frown}{FG}$.



Solution

Because $\overline{FG}$ and $\overline{JK}$ are congruent chords in congruent circles, the corresponding minor arcs $\overset{\frown}{FG}$ and $\overset{\frown}{JK}$ are congruent.

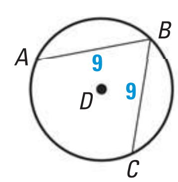
▶ So, $m\overset{\frown}{FG} = m\overset{\frown}{JK} = 80°$.

✓ GUIDED PRACTICE for Example 1

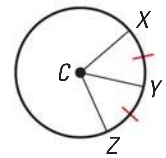
Use the diagram of $\odot D$.

1. If $m\overset{\frown}{AB} = 110°$, find $m\overset{\frown}{BC}$.

2. If $m\overset{\frown}{AC} = 150°$, find $m\overset{\frown}{AB}$.



BISECTING ARCS If $\overset{\frown}{XY} \cong \overset{\frown}{YZ}$, then the point Y, and any line, segment, or ray that contains Y, bisects $\overset{\frown}{XYZ}$.



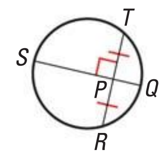
$\overline{CY}$ bisects $\overset{\frown}{XYZ}$.

THEOREMS *For Your Notebook*

THEOREM 10.4

If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.

If $\overline{QS}$ is a perpendicular bisector of $\overline{TR}$, then $\overline{QS}$ is a diameter of the circle.

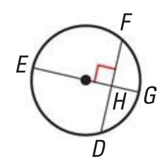


Proof: Ex. 31, p. 670

THEOREM 10.5

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

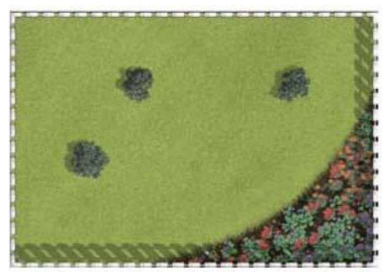
If $\overline{EG}$ is a diameter and $\overline{EG} \perp \overline{DF}$, then $\overline{HD} \cong \overline{HF}$ and $\overset{\frown}{GD} \cong \overset{\frown}{GF}$.



Proof: Ex. 32, p. 670

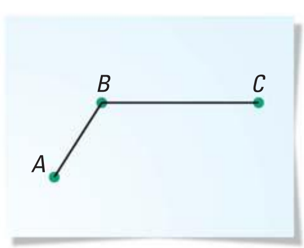
EXAMPLE 2 Use perpendicular bisectors

GARDENING Three bushes are arranged in a garden as shown. Where should you place a sprinkler so that it is the same distance from each bush?



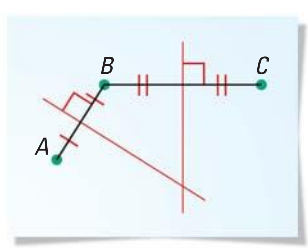
Solution

STEP 1



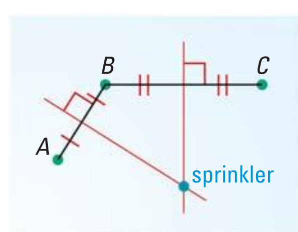
Label the bushes A, B, and C, as shown. Draw segments $\overline{AB}$ and $\overline{BC}$.

STEP 2



Draw the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$. By Theorem 10.4, these are diameters of the circle containing A, B, and C.

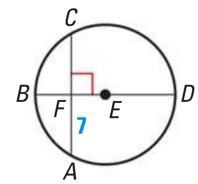
STEP 3



Find the point where these bisectors intersect. This is the center of the circle through A, B, and C, and so it is equidistant from each point.

EXAMPLE 3 Use a diameter

Use the diagram of $\odot E$ to find the length of $\overline{AC}$. Tell what theorem you use.

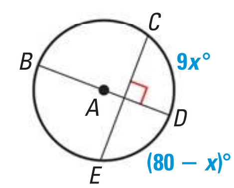


Solution

Diameter $\overline{BD}$ is perpendicular to $\overline{AC}$. So, by Theorem 10.5, $\overline{BD}$ bisects $\overline{AC}$, and $CF = AF$. Therefore, $AC = 2(AF) = 2(7) = 14$.

✓ GUIDED PRACTICE for Examples 2 and 3

Find the measure of the indicated arc in the diagram.

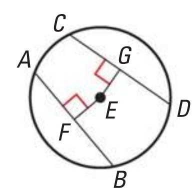


3. $\overset{\frown}{CD}$ **4.** $\overset{\frown}{DE}$ **5.** $\overset{\frown}{CE}$

THEOREM *For Your Notebook*

THEOREM 10.6

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

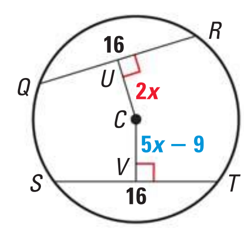


$\overline{AB} \cong \overline{CD}$ if and only if $EF = EG$.

Proof: Ex. 33, p. 670

EXAMPLE 4 Use Theorem 10.6

In the diagram of $\odot C$, $QR = ST = 16$. Find CU.



Solution

Chords $\overline{QR}$ and $\overline{ST}$ are congruent, so by Theorem 10.6 they are equidisant from C. Therefore, $CU = CV$.

$CU = CV$	**Use Theorem 10.6.**
$2x = 5x - 9$	**Substitute.**
$x = 3$	**Solve for *x*.**

▶ So, $CU = 2x = 2(3) = 6$.

✓ GUIDED PRACTICE for Example 4

In the diagram in Example 4, suppose $ST = 32$, and $CU = CV = 12$. Find the given length.

6. QR **7.** QU **8.** The radius of $\odot C$

10.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 9, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 22, and 26

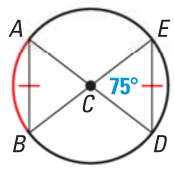
SKILL PRACTICE

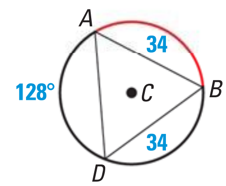
1. **VOCABULARY** *Describe* what it means to *bisect* an arc.

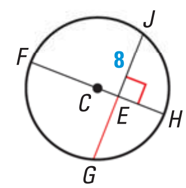
2. ★ **WRITING** Two chords of a circle are perpendicular and congruent. Does one of them have to be a diameter? *Explain* your reasoning.

EXAMPLES 1 and 3 on pp. 664, 666 for Exs. 3–5

FINDING ARC MEASURES Find the measure of the red arc or chord in ⊙*C*.

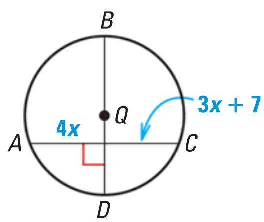
3. 

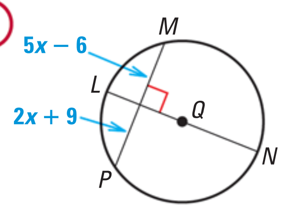
4. 

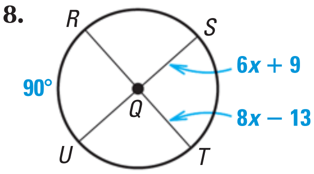
5. 

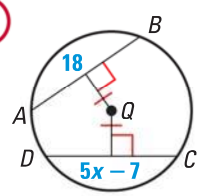
EXAMPLES 3 and 4 on p. 666 for Exs. 6–11

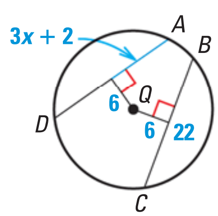
xy ALGEBRA Find the value of *x* in ⊙*Q*. *Explain* your reasoning.

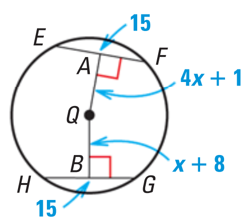
6. 

7. 

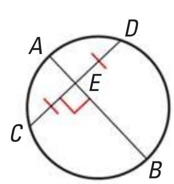
8. 

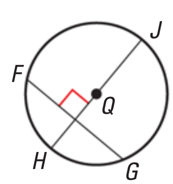
9. 

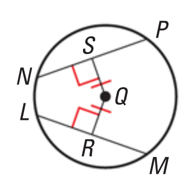
10. 

11. 

REASONING In Exercises 12–14, what can you conclude about the diagram shown? State a theorem that justifies your answer.

12. 

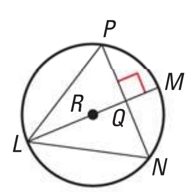
13. 

14. 

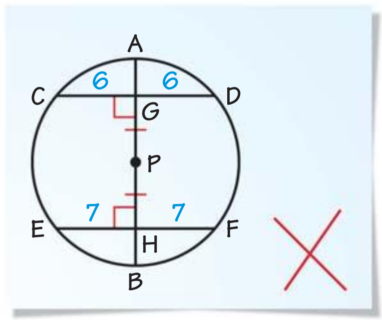
15. ★ **MULTIPLE CHOICE** In the diagram of ⊙*R*, which congruence relation is not necessarily true?

   (A) $\overline{PQ} \cong \overline{QN}$ (B) $\overline{NL} \cong \overline{LP}$

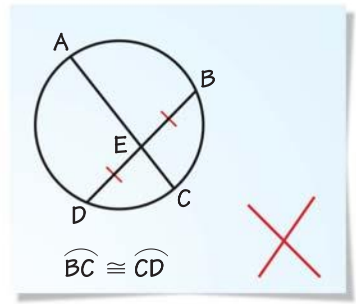
   (C) $\overset{\frown}{MN} \cong \overset{\frown}{MP}$ (D) $\overline{PN} \cong \overline{PL}$



16. **ERROR ANALYSIS** *Explain* what is wrong with the diagram of $\odot P$.

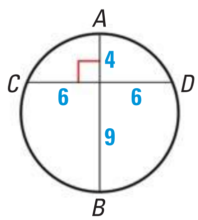


17. **ERROR ANALYSIS** *Explain* why the congruence statement is wrong.

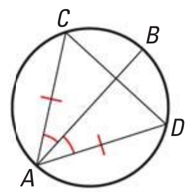


IDENTIFYING DIAMETERS **Determine whether $\overline{AB}$ is a diameter of the circle. *Explain* your reasoning.**

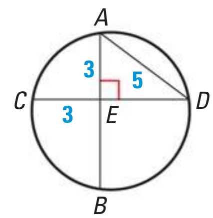
18.



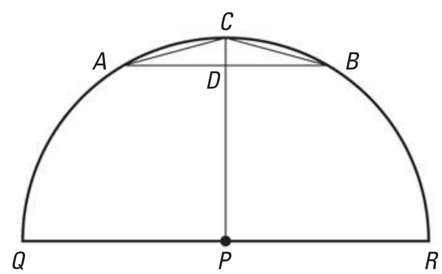
19.



20.



21. **REASONING** In the diagram of semicircle $\overset{\frown}{QCR}$, $\overline{PC} \cong \overline{AB}$ and $m\overset{\frown}{AC} = 30°$. *Explain* how you can conclude that $\triangle ADC \cong \triangle BDC$.

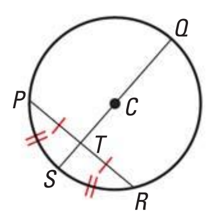


22. ★ **WRITING** Theorem 10.4 is nearly the converse of Theorem 10.5.

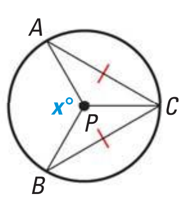
a. Write the converse of Theorem 10.5. *Explain* how it is different from Theorem 10.4.

b. Copy the diagram of $\odot C$ and draw auxiliary segments $\overline{PC}$ and $\overline{RC}$. Use congruent triangles to prove the converse of Theorem 10.5.

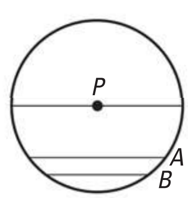
c. Use the converse of Theorem 10.5 to show that $QP = QR$ in the diagram of $\odot C$.



23. **xy ALGEBRA** In $\odot P$ below, $\overline{AC}$, $\overline{BC}$, and all arcs have integer measures. Show that x must be even.

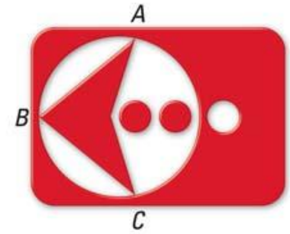


24. **CHALLENGE** In $\odot P$ below, the lengths of the parallel chords are 20, 16, and 12. Find $m\overset{\frown}{AB}$.



PROBLEM SOLVING

25. **LOGO DESIGN** The owner of a new company would like the company logo to be a picture of an arrow inscribed in a circle, as shown. For symmetry, she wants $\overset{\frown}{AB}$ to be congruent to $\overset{\frown}{BC}$. How should $\overline{AB}$ and $\overline{BC}$ be related in order for the logo to be exactly as desired?



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EXAMPLE 2 on p. 665 for Ex. 26

26. ★ **OPEN-ENDED MATH** In the cross section of the submarine shown, the control panels are parallel and the same length. *Explain* two ways you can find the center of the cross section.

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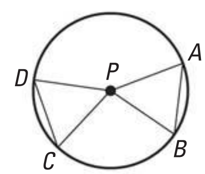
PROVING THEOREM 10.3 **In Exercises 27 and 28, prove Theorem 10.3.**

27. **GIVEN** ▶ $\overline{AB}$ and $\overline{CD}$ are congruent chords.
PROVE ▶ $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

28. **GIVEN** ▶ $\overline{AB}$ and $\overline{CD}$ are chords and $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.
PROVE ▶ $\overline{AB} \cong \overline{CD}$



29. **CHORD LENGTHS** Make and prove a conjecture about chord lengths.

a. Sketch a circle with two noncongruent chords. Is the *longer* chord or the *shorter* chord closer to the center of the circle? Repeat this experiment several times.

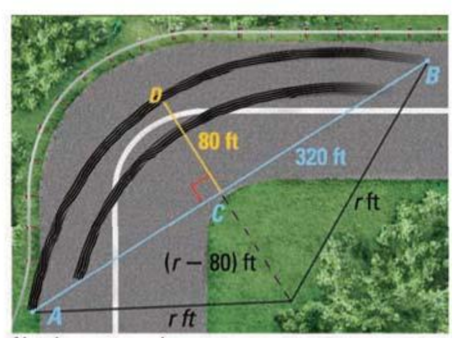
b. Form a conjecture related to your experiment in part (a).

c. Use the Pythagorean Theorem to prove your conjecture.

30. **MULTI-STEP PROBLEM** If a car goes around a turn too quickly, it can leave tracks that form an arc of a circle. By finding the radius of the circle, accident investigators can estimate the speed of the car.

a. To find the radius, choose points A and B on the tire marks. Then find the midpoint C of $\overline{AB}$. Measure $\overline{CD}$, as shown. Find the radius r of the circle.

b. The formula $S = 3.86\sqrt{fr}$ can be used to estimate a car's speed in miles per hours, where f is the *coefficient of friction* and r is the radius of the circle in feet. The coefficient of friction measures how slippery a road is. If $f = 0.7$, estimate the car's speed in part (a).



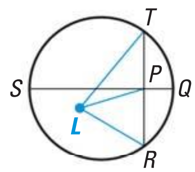
Not drawn to scale

PROVING THEOREMS 10.4 AND 10.5 **Write proofs.**

31. **GIVEN ▶** $\overline{QS}$ is the perpendicular bisector of $\overline{RT}$.

PROVE ▶ $\overline{QS}$ is a diameter of $\odot L$.

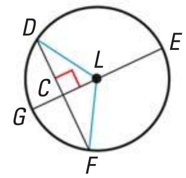
Plan for Proof Use indirect reasoning. Assume center L is not on $\overline{QS}$. Prove that $\triangle RLP \cong \triangle TLP$, so $\overline{PL} \perp \overline{RT}$. Then use the Perpendicular Postulate.



32. **GIVEN ▶** $\overline{EG}$ is a diameter of $\odot L$. $\overline{EG} \perp \overline{DF}$

PROVE ▶ $\overline{CD} \cong \overline{CF}$, $\overset{\frown}{DG} \cong \overset{\frown}{FG}$

Plan for Proof Draw $\overline{LD}$ and $\overline{LF}$. Use congruent triangles to show $\overline{CD} \cong \overline{CF}$ and $\angle DLG \cong \angle FLG$. Then show $\overset{\frown}{DG} \cong \overset{\frown}{FG}$.



33. **PROVING THEOREM 10.6** For Theorem 10.6, prove both cases of the biconditional. Use the diagram shown for the theorem on page 666.

34. **CHALLENGE** A car is designed so that the rear wheel is only partially visible below the body of the car, as shown. The bottom panel is parallel to the ground. Prove that the point where the tire touches the ground bisects $\overset{\frown}{AB}$.

MIXED REVIEW

PREVIEW
Prepare for Lesson 10.4 in Exs. 35–37.

35. The measures of the interior angles of a quadrilateral are 100°, 140°, $(x + 20)°$, and $(2x + 10)°$. Find the value of x. *(p. 507)*

Quadrilateral *JKLM* is a parallelogram. Graph ▱*JKLM*. Decide whether it is best described as a *rectangle*, a *rhombus*, or a *square*. *(p. 552)*

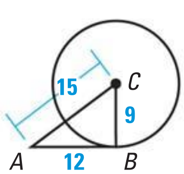
36. $J(-3, 5)$, $K(2, 5)$, $L(2, -1)$, $M(-3, -1)$

37. $J(-5, 2)$, $K(1, 1)$, $L(2, -5)$, $M(-4, -4)$

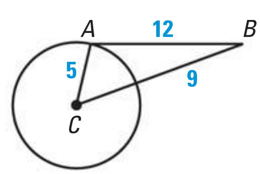
QUIZ *for Lessons 10.1–10.3*

Determine whether $\overline{AB}$ is tangent to $\odot C$. *Explain* your reasoning. *(p. 651)*

1.



2.



3. If $m\overset{\frown}{EFG} = 195°$, and $m\overset{\frown}{EF} = 80°$, find $m\overset{\frown}{FG}$ and $m\overset{\frown}{EG}$. *(p. 659)*

4. The points A, B, and D are on $\odot C$, $\overline{AB} \cong \overline{BD}$, and $m\overset{\frown}{ABD} = 194°$. What is the measure of $\overset{\frown}{AB}$? *(p. 664)*